UK Junior Mathematical Olympiad 1999

Organised by The United Kingdom Mathematics Trust

Tuesday 8th June 1999

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators *and measuring instruments* is forbidden.**
3. All candidates must be in *School Year 8 or below* (English and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required.* Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left comer with the Front Sheet on top.

   Do not hand in rough work.
5. Questions A 1-10 are relatively short questions. Try to complete Section A within the first hour. Then allow at least one hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that your answer must be accompanied by clear explanations and proofs.
   Work in rough first, then lay out your final solution giving clear explanations of each step.
7. These problems are meant to be hard! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate should aim to do most of Section A and give solutions to at least two questions in Section B.
8. Numerical answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

A1 What is the angle between the hands of a clock at 4.20 p.m.?

A2 In how many different ways can 50 be written as the sum of two prime numbers ?
(Note: $x + y$ and $y + x$ do not count as different.)

A3 Today's date, 8/6/99, might be read as a fraction in two different ways:

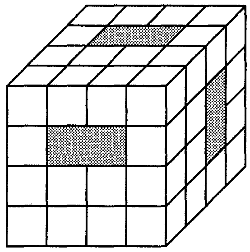
$$\frac{^8/_6}{99} \quad \text{or} \quad \frac{8}{^6/_{99}}.$$

Find the whole number nearest to the sum of these two fractions.

A4 Tickets for a concert cost £7 for an adult and £5 for a child.
The organiser of a group works out that the tickets for the group will cost a total of £108, and notices that the cost would be the same if the prices were changed to £8 per adult and £4 per child.
How many children are in the group?

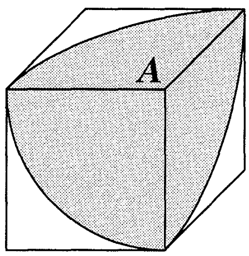
A5 UKMT, TMUK and KTUM are all different arrangements of the letters U, K, M and T. If the number of all the different arrangements of these four letters is p and the number of all the different arrangements of the letters U, K, J, M and O is q, what is the value of $\frac{q}{p}$?

A6 A cube is made of 64 small cubes. Three holes are made, with each hole perpendicular to two faces and passing right through the cube. The shape and position of each hole is shown in the diagram.
How many small cubes are in the remaining solid?



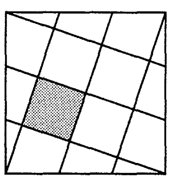
A7 Before the decimalisation of money in the UK, there were 12 pence (d) in 1 shilling (s) and 20 shillings in 1 pound (£). Thus 1 pound 3 shillings and 4 pence was written £1 3s 4d. What would have been the total cost of 7 items each costing £1 6s 8d? Write your answer in simplest £ s d form.

A8 At a corner, A, of the cube shown here, a circular arc with centre A is drawn on each of the three faces meeting at the corner A.
What fraction of the surface area of the cube is shaded?



A9 Skimmed milk contains 0.1% fat and pasteurised whole milk contains 4% fat. When 6 litres of skimmed milk are mixed with n litres of pasteurised whole milk, the fat content of the resulting mixture is 1.66%.
What is the value of n?

A10 What fraction of the whole square is occupied by the shaded square?



Section B

Your solutions to Section B will have a major effect on the JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1 Suppose you know that the middle two digits of a four digit integer N are '12' in that order and that N is an exact multiple of 15.

Determine all the different possibilities for the integer N.

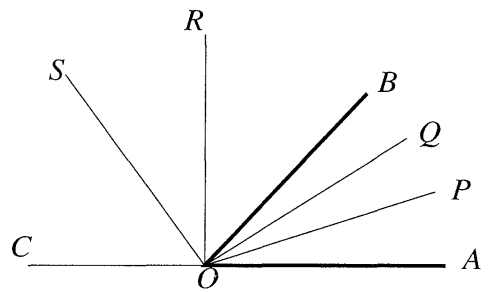
(You must explain clearly why your list is complete.)

B2 AOC is a straight line and angle $AOB = 42°$. OP and OQ trisect angle AOB (which means they divide the angle into three equal parts).

OR and OS trisect angle BOC.

(a) Showing all working, calculate angle QOR and angle POS.

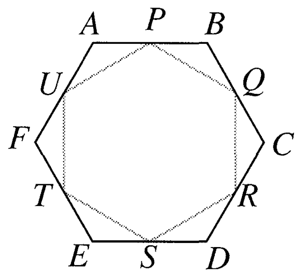
(b) Calculate QOR when angle $AOB = x°$.



B3 Alice, the March Hare and the Mock Turtle were the only three competitors at the Wonderland sports day, and all three of them competed in each event. The scoring system was exactly the same for each event: the points awarded for first, second and third places were all positive integers and (even in Wonderland) more points were awarded for first place than for second and more points for second place than for third.

Of course, the March Hare won the Sack Race. At the end of the day, Alice had scored 18 points while the Mock Turtle had 9 points and the March Hare had 8 points. Can you decide how many events there were? And can you tell who came last in 'Egg and Spoon' race?

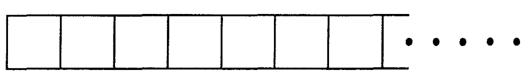
B4 The regular hexagon $ABCDEF$ has sides of length 2. The point P is the midpoint of AB, Q is the midpoint of BC, and so on. Find the area of the hexagon $PQRSTU$.

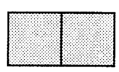


B5 (a) Find all the two-digit numbers which are increased by 75% when their digits are reversed.

(b) Find all the three-digit numbers which are increased by 75% when their digits are reversed.

B6





A counter.

Two players, X and Y, play a game on a board which consists of a narrow strip which is one square wide and n squares long. They take turns at placing counters, which are one square wide and two squares long, on unoccupied squares on the board. The first player who cannot go loses. X always plays first and both players always make the best available move.

(a) Who wins the game on a 4×1 board? Explain how they must play to win and why they are then certain to win.

(b) Who wins the game on a 5×1 board? Explain why. So who wins on a 7×1 board? Explain.

(c) Who wins the game on a 6×1 board? How?

(d) Who wins the game on an 8×1 board? How?